

## Blind Watermarking Algorithm Based on Wavelet-Transform & Fractal Self-Similarity

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### Abstract

*An algorithm for embedding a meaningful 2-bit watermark image in carrier image is proposed by using Zigzag image scrambling based on discrete wavelet transform (DWT) and fractal self-similarity, along the Hilbert fractal curve scan sequence in the intermediate frequency subimage of wavelet decomposition of a carrier image, according to the mask template of Sierpinski carpet. Extracting the embedded watermark does not need the original carrier image and the original watermark image, which is a blind watermark algorithm. By using fractal self-similarity and Sierpinski carpet mask, watermark information can be embedded into the non-central parts of the carrier image repeatedly  $N$  times. The algorithm has shown the higher watermark imperceptibility and robustness under cutting and falsifying attacks of image, which is proved to be true by experimental results. The algorithm can be applied in DRM to protect digital image copyright.*

**Keywords:** *Wavelet Transform, Fractal Self-Similarity, Hilbert Curve, Sierpinski Carpet, Blind Watermark, Digital Rights Management*

## 1. Introduction

Digital Rights Management (DRM) can manage and protect the rights of digital contents of all participants, in the entire value chain from production distribution to consumption [1, 2]. DRM is a systemic concept that contains management, transmission and distribution of copyrighted digital media content as well as the use of digital copyright information. DRM provides a higher security for copyright works, with Digital watermark as one of the protection techniques commonly used up to now [3]. Digital watermark is a technique to provide copyright protection, secret communications, authenticity's identification of data file and product marking, by replacing the redundant data with the copyright information-digit, serial number, text, image sign and others- in digital works. This is called copyright embedding [4,5]. In the digital copyright protection, the embedded copyright information can be taken as evidence to prove the fact that who owns the ownership of a digital product in court, so as to protect the owner's copyright through embedding and extracting the copyright in digital products by watermark technology.

Wavelet transformation leads to a decomposition of an image in four components: the approximation coefficients sub-image  $cA$  and the details coefficients sub-images  $cH$ ,  $cV$ , and  $cD$  (horizontal, vertical, and diagonal), among which  $cA$  represents low frequency component,  $cH$ ,  $cV$ , and  $cD$  represent high frequency component respectively on corresponding resolutions [6,7,8].

High-frequency sub-images [ $cH$ ,  $cV$ , and  $cD$ ] show the texture details of the original image, while low-frequency sub-image  $cA$  means the  $1/4$  approximation of the original image. Since human visual system is sensitive to the low frequency part of the noise relatively, the transparency (namely concealment) of the watermark embedded in the low frequency part  $cA$  is poor, although it has stronger resistance power against foreign attacks. To make the watermark concealed adequately, the watermark should be embedded in higher frequency range. However, the most energy of a image is usually concentrated to the low frequency range, the embedded watermark in high-frequency is easy to get lost due to different attacks such as external noise, quantization and conventional image processing. In order to solve the problem of watermark embedding contradiction caused by low and high frequency disability, the watermark is embedded into the high-band  $cH$  of 1-layer wavelet decomposition of the carrier image, and not embedded into the high frequency band of 2-layer wavelet decomposition,

which is actually embedded into the relative medium frequency band of the carrier image of Wavelet decomposition.

Fractal graph has a property of self-similarity. The self-similarity exists between one part and another part, or the part and the whole, of it [9]. All the pixels of a  $(4 \times 2^n) \times (4 \times 2^n)$ -order image can be traversed only once along a Hilbert curve scan sequence. By using fractal self-similarity, along Hilbert curve scanning sequence of the carrier image, the watermarking can be embedded in carrier image for  $M$  times.

The Sierpinski carpet is a plane fractal first described by Waclaw Sierpiński in 1916. The carpet is a generalization of the Cantor set to two dimensions (another is Cantor dust). Human visual system often focus more on the central part of the visual object; With the transfer of sight line, the more attention is also focused on the central part of sub-section of the visual object. The process and the phenomenon can be approximately simulated by the Sierpinski carpet.

A new watermark algorithm is proposed. Firstly, a meaningful 2-bit watermark image is scrambled  $N$  times by Zigzag in order to make its energy uniformly distribute in its entire space. Then, the scrambled watermark sequence is embedded into the carrier image for  $M$  times along the Hilbert curve scan sequence according to the characteristic of the carrier image and using the Sierpinski carpet as a watermark-embedding mask.

The watermark detection does not depend on the original carrier image, and the watermark image is extracted by inverse Zigzag Coding transform on the detection information  $N$  times. The experiment indicates that the algorithm on attacks of image cutting has strong robustness, and good watermark invisibility. This algorithm can be applied in DRM to protect digital image copyright.

The remainder of the paper is organized as following: In Section 2, the principles of algorithm is introduced including Wavelet Transform, Fractal and Self-Similarity as well as Zigzag encoding digital image scrambling method. Embedding and Extracting of Watermark is described in Section 3. Finally, we give the experiments and its performance analysis by simulation on Matlab 2009a in Section 4. In Section 5, we conclude the paper and give an outlook to our future work.

## 2. Principles of Algorithm

Wavelet Transform theory, fractal self-similarity and Zigzag image scrambling technology are used in the algorithm.

### 2.1. Wavelet Transform [8]

The approximation coefficients matrix  $cA$  and the details coefficients matrices  $cH$ ,  $cV$ , and  $cD$  (horizontal, vertical, and diagonal) are obtained by wavelet decomposition of Image  $A$ . Here,  $cA$  represents low frequency component,  $cH$ ,  $cV$ , and  $cD$  are represents high frequency component respectively under corresponding resolution. Because of wavelet base orthogonality, image wavelet decomposition does not produce redundant data. This way, the properties of different frequency band signals can be easily analyzed in frequency domains.

Mallat rapid algorithm has supplied a suitable implementation framework for discrete multi-wavelet transform. However, when using the vector filter group to implement discrete multi-wavelet transformation, because vector filter group only can process the vector signal, the image must be transformed into vector image ( $r \times r$  the matrix block) firstly before the image is decomposed by the multi-wavelet. This step is completed by the pre-filter, is called the pre-filter. Firstly, the input scalar image  $f(n)$  is executed the critical sampling to obtains the vector

$$x(k) = [f(rk) \ \cdots \ f(rk + r - 1)]^T$$

Then, let  $x(k)$  be filtered by the  $r \times r$  pre-filter  $Q(\omega)$  and obtain initial vector signal  $C_k^0$  which needs to be decomposed by multi-wavelet. If the size of the original image is  $M \times N$  that is input into the pre-filter, the size of the image is  $(M \times N) / r^2$  that is output from the pre-filter. Correspondingly after the image signal is reconstructed by multi-wavelet, it must be

performed post-filtering by post-filter  $P(\omega)$ , and then  $C_k^0$  can be recovered to the original state  $f(n)$ , which the post-filter is the inverse pre-filter.

According to two-dimension Mallat fast wavelet transform algorithm,  $D_j^i$  (3 of the high components) and  $C^j$  (a low frequency component) are generated under the  $n$ th wavelet decomposition of image [10].  $D_j^i$  ( $i=H, V$  and  $D$ ) stands for the horizontal, vertical, and diagonal details (the high frequency components) at level  $j$ ; Subscript  $m$  and  $n$  stand for corresponding row and column in image; Subscript  $j$  stands for wavelet decomposition scale.

Mallat wavelet decomposition formula is

$$\begin{cases} C_j = H_m H_n C_{j-1} \\ D_j^H = G_m H_n C_{j-1} \\ D_j^V = H_m G_n C_{j-1} \\ D_j^D = G_m G_n C_{j-1} \end{cases} \quad (1)$$

And the relevant wavelet recomposition formula is

$$\begin{aligned} C_{j-1} = & H_m^* H_n^* C_j + H_m^* G_n^* D_j^H \\ & + G_m^* H_n^* D_j^V + G_m^* G_n^* D_j^D \end{aligned} \quad (2)$$

Among them,  $H^*$ 、 $G^*$  denote the conjugate transposed matrix of  $H$  and  $G$  respectively.

## 2.2. Fractal and Self-Similarity

An object is self-similar only the object can be broken down into an arbitrary number of small pieces, and each of those pieces is a replica of the entire structure [9]. The fractal basic idea is that any objective matter represents self-similar structure. That is to say, there is a certain similarity between a part of the fractal and the entire fractal itself in their form, function, information, time, space, etc on the statistical significance. This phenomenon is called the self-similarity. Hilbert Curve is used as the watermark embed-path and the Sierpinski Carpet of order 4 is used as the watermark embed-mask.

### 2.2.1. Hilbert Curve

A German mathematician, Hilbert, constructed Hilbert curve [11] in 1891, All the points of a square can be traversed only once along a Hilbert curve scan sequence.

For example, a  $8 \times 8$  Hilbert curve in Figure 1(b) contains four  $4 \times 4$  Hilbert curves in Figure 1(a) and a  $16 \times 16$  Hilbert curve in Figure 1(c) contains four  $8 \times 8$  Hilbert curves in Figure 1(b). It means that every part of the Hilbert curve has a similar structure with the entire curve. No matter how much it get zoomed in or zoomed out, the structure of Hilbert curve remains the same.

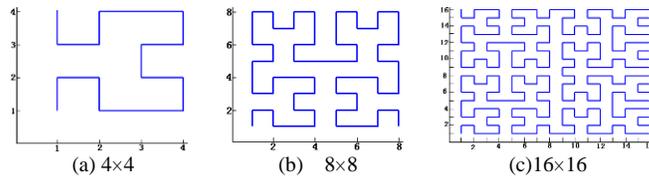
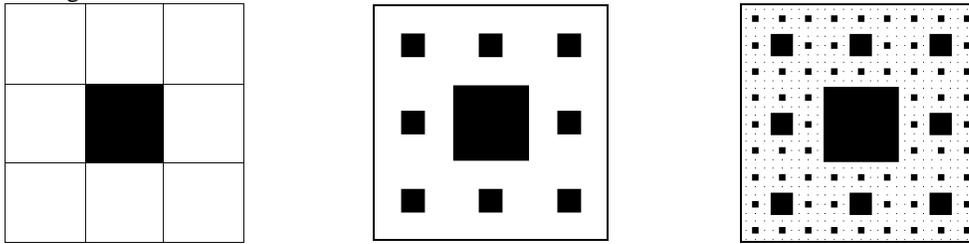


Figure 1. Hilbert Curves

### 2.2.2. Sierpinski Carpet

The Sierpinski carpet is produced from a square graph by iteration way, its iteration rule is: The construction of the Sierpinski carpet begins with a square, the square is cut into 9 congruent subsquares in a 3-by-3 grid, and the central subsquare is removed. The same procedure is then applied recursively to the remaining 8 subsquares, ad infinitum.

The Figure 2 below shows 3 of the first four iterations.



(a) Sierpinski carpet of order 1      (b) Sierpinski carpet of order 2      (c) Sierpinski carpet of order 4

**Figure 2.** Sierpinski carpet

The Hausdorff dimension of the carpet is  $\log 8 / \log 3 \approx 1.8928$ .

The area of the Sierpinski carpet is calculated as the formula:

$$A(0) = 1, \quad \text{Initial area}$$

$$A(n) = (8/9)^n, \quad n \geq 0, \quad A(n): \text{the area of Sierpinski carpet of order } n$$

The area of the carpet is zero (in standard Lebesgue measure).

Human visual system often focus more on the central part of the visual object; With the transfer of sight line, the more attention is also focused on the central part of sub-section of the visual object. The process and the phenomenon can be approximately simulated by the Sierpinski carpet.

The area of 4-order Sierpinski carpet is about 0.62. 4-order Sierpinski carpet is adopted as the template of watermark embedding. The watermark information is not embedded into the pixels of the carrier image which pixel's positions are respectively corresponding to the value 0 pixel's positions of the Sierpinski carpet, in order to maintain the original information of the special parts of the carrier image; Contrastively, a certain rules is adopted to determine whether the watermark information is embedded into the pixels of the carrier image which pixel's positions are respectively corresponding to the value 1 pixel's positions of the Sierpinski carpet.

Thus about 62% pixels of the carrier image participate in the watermark information embedding. Contrastively, the central and sub-central parts (about 38% pixels) of the carrier image do not participate in the watermark information embedding. It can ensure the image quality of the central and sub-central parts of the watermark-embedded image as a result of the watermark information is not embedded into the central and sub-central parts of the carrier image, and it can ensure the sufficient watermark information can also be embedded.

### 2.3. Zigzag encoding digital image scrambling method

A digital image can be defined by a matrix of "pixels". The row index and column index of the every element of the matrix are the coordinate of each pixel of the image displayed on the computer screen. The value of the element of the matrix is the gray value of the corresponding pixel. A new image matrix is obtained by changing the position or the gray value of the element of an image matrix. This is called image scrambling. The image scrambling based on Zigzag encoding in Figure 3 is adopted to realize the invisibility of watermark information.

In this way, a matrix is changed into a  $1 \times 64$  vector. The vector is rearranged and generates a new  $8 \times 8$  matrix as shown in Figure 4.

A  $64 \times 64$  meaningful binary watermark image  $W$  is scrambled for  $N$  times according to Zigzag encoding. Then, a desultory and meaningless image is obtained with eliminating the

spatial correlation of watermark image pixels. Here N can be regarded as the encryption key of watermark image.

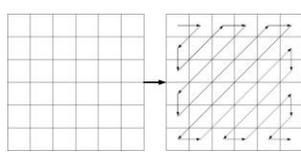


Figure 3. Coefficient Re-Order

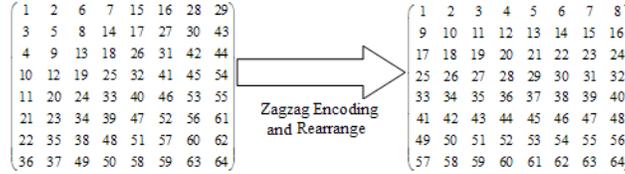


Figure 4. Zigzag Encoding

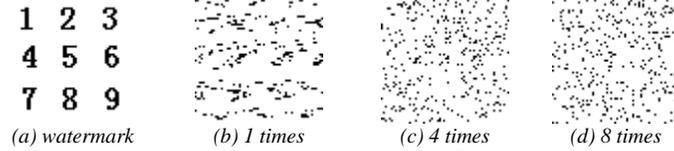


Figure 5. Watermark image Scrambling

In Figure 5, 64×64-pixel (a) is regarded as a watermark image, (b) is scrambled 1 times of (a), (c) is scrambled 4 times of (a), (d) is scrambled 8 times of (a). It can be concluded that the more the scrambling times, the more imperceptibility of the watermark. This results in more robustness of embedded watermark when suffering the crack attack.

### 3. Embedding and Extracting of Watermark

The watermark is embedded into the medium frequency component of 1-level wavelet decomposition of a carrier image along the Hilbert curve sequence and according to the mask template of Sierpinski carpet, and a new watermark-embedded image is obtain by inverse wavelet transform.

#### 3.1. Embedding Algorithm

A 256 x 256 gray carrier image I and a 64x 64 binary watermark image W is used. Watermark embedding process is as follows.

(1) According to Zigzag coding rules, the watermark image W is scrambled for N times, and the scrambled watermark image  $W_r = \{w(i,j) | w(i,j) \in \{0,1\}\}$  is obtained.

(2) The  $[A1, H1, V1, D1]$  are obtained by 1-layer harr wavelet decomposition of carrier Image I. The subimage H1 has 128 x128 pixels,

(3) The watermark  $W_r$  with 4096 bit, distributed into H1 with 16,384 pixels, can be embedded for 4 times.

(4) According to Hilbert fractal curve scanning sequence and the mask template of Sierpinski carpet, the digital watermark image  $W_r$  is embedded into subimage H1 for 4 times.

The elements of subimage H1 are divided into pixel-pairs, comparing the gray value of  $p_i$  and  $p_j$  in each pixel-pair  $(p_i, p_j)$  in H1. If the former  $p_i$  is greater than the latter  $p_j$ ,  $(p_i, p_j)$  denotes 1; if the former  $p_i$  is less than the latter  $p_j$ ,  $(p_i, p_j)$  denotes 0; if  $p_i$  is equal to  $p_j$ , and one of the values of  $(p_i, p_j)$  will make minor adjustment. This makes the less-than or greater-than relationship between two elements of each pixel corresponding to each pixel of the binary watermark image. If the corresponding relationship is not exists, the values of two elements of the each pixel-pair exchanged mutually.

(5) The watermark image IW is obtained by IDWT on  $[A1, H1, V1, \text{and } D1]$  of embedded watermark.

#### 3.2. Watermark Extraction

(1) Firstly, the carrier image with a watermark is decomposed by wavelet transform in level 1.

$[A1, H1, V1, D1]=dwt2(f, 'haar')$

Here, matrix  $f$  denotes a carrier gray image with embedded watermark, the size of  $256 \times 256$ , the resolution of  $256 \times 256$ , and 256 gray levels.

(2) Then, the watermark is extracted from  $H1$ , which is the reverse process of the watermark embedding. The  $H1$  is divided into pixel-pair. Comparing the value of each pixel-pair, if the former greater than the latter, extract 1; If the former less than the latter, extract 0.

The process of watermark extraction is as follows:

$I_w$  denotes the embedded-watermark image,  $S = \{s_{i,j}, s_{i,j} \in \{0,1\}\}$  denotes the extraction information of watermark,  $N$  denotes the key that is the number of times of watermark scrambling.

(1)  $I_w$  is transformed to obtain  $[A1, H1, V1, \text{and } D1]$  by DWT.

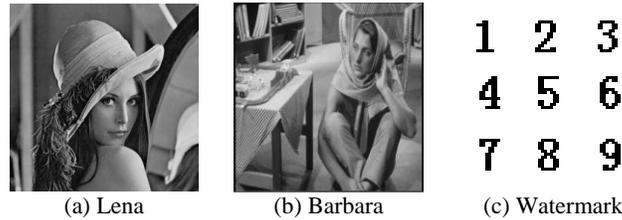
(2) Extract the watermark information from the  $H1$  that the watermark information is embedded in.

(3) Extract the watermark information along the Hilbert curve scan sequence by repeating the above step 2, according to the mask template of Sierpinski carpet.

(4) Restore one-dimensional watermark sequence  $S = \{s_{i,j}, s_{i,j} \in \{0,1\}\}$  to the 2-dimensional scrambled image  $W_s$ . Then, the decrypted watermark information is extracted out by scrambling the image  $W_s$  for  $N$  times inversely according to the rule of anti-Zigzag coding.

#### 4. Experiments

In Figure 6, Gray images of Lena (a) ( $256 \times 256 \times 8$ ) and Barbara (b) ( $512 \times 512 \times 8$ ) are used as the carrier images, and a  $64 \times 64$  binary image (c) is used as a watermark image.



**Figure 6.** Carrier Images and Watermark Image

**Table 1.** Anti-attack Experiment of Watermark

Attack Method \ Index	Lena		Barbara	
	PSNR(dB)	NCC	PSNR(dB)	NCC
No Attacking	33.7608	0.9989	39.8213	0.9998
Cutting 1/8	15.6433	0.9992	12.9241	0.9230
Cutting 1/4	9.7387	0.9992	11.0198	0.9105
Cutting 1/2	6.0792	0.9784	7.4305	0.8778
Malicious falsifying	16.9760	0.9653	22.0009	0.9864

**Table 2.** Anti-attack Experiment of Watermark Adopted Sierprinski's Carpet Mask

Attack Method \ Index	Lena		Barbara	
	PSNR(dB)	NCC	PSNR(dB)	NCC
No Attacking	35.1312	0.9992	40.2693	0.9998
Cutting 1/8	15.6615	0.9448	12.9246	0.9230
Cutting 1/4	9.7423	0.8793	11.0198	0.9105
Cutting 1/2	6.0803	0.8262	7.4305	0.8778
Malicious falsifying	16.9806	0.9468	22.0080	0.9864

The imperceptibility of watermark embedding algorithm is quantitatively expressed by the PSNR (peak signal-to-noise ratio) between the embedded-watermark image and the original carrier image, and the robustness of watermark embedding algorithm is quantitatively expressed by the NCC

(normalized cross-correlation) between the embedded-watermark image and the original carrier image. The larger the PSNR is, the higher the imperceptibility would be. And the larger the NCC is, the higher the robustness would be [12].

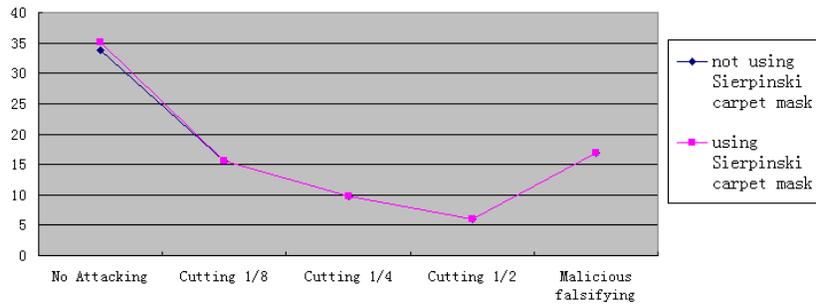


Figure 7.1 Comparison between (Lena) PSNRs

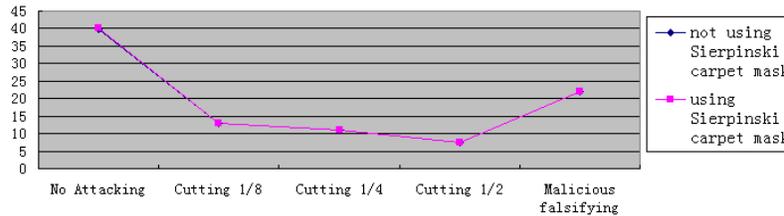


Figure 7.2 Comparison between (Barbara) PSNRs

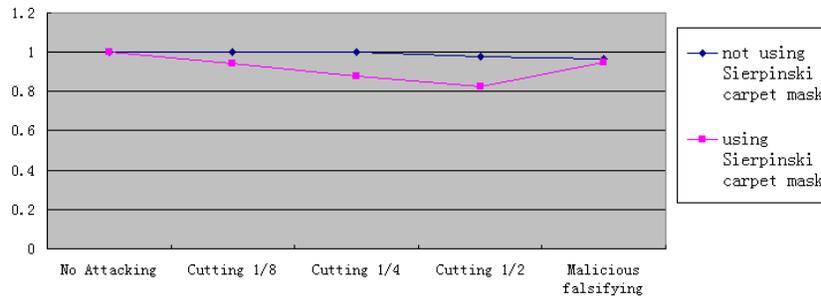


Figure 8.1. Comparison between (Lena) NCCs

The experimental results shows that the PSNRs of watermarked image of Lena and Barbara are 33.7608dB and 39.8213dB respectively in Table 1, 35.1312dB and 40.2693dB respectively in Table 2. From the data comparisons of Table 1, Table 2 and Figure 7, it can be concluded that the watermark's imperceptibility with using Sierpinski carpet mask is higher than one without Sierpinski carpet mask; Furthermore, the robustness differences between using and not using Sierpinski carpet mask are very slight as a conclusion of Figure 8.1 and Figure 8.2.



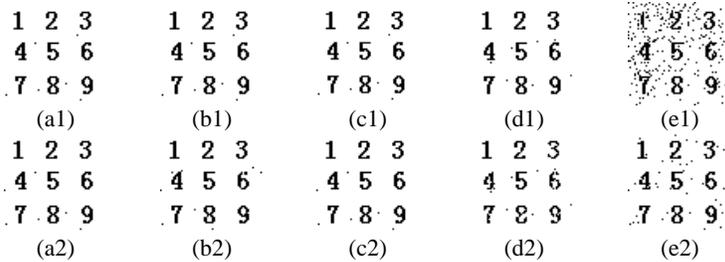
Figure 8.2. Comparison between (Barbara) NCCs

In Figure 9, image (a1) is the watermark-embedded image without using Sierpinski carpet mask, (b1) is (a1) that is cut off 1/8 on the top-left, (c1) is (a1) that is cut off 1/4 on the top-left, (d1) is (a1) that is cut off 1/2 on the left, (e1) is (a1) that is maliciously falsified in the center by another image.

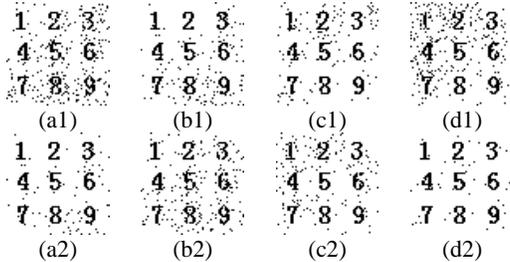
In Figure 9, image (a2) is the watermark-embedded image with using Sierpinski carpet mask, (b2) is (a2) that is cut off 1/8 on the top-left, (c2) is (a2) that is cut off 1/4 on the top-left, (d2) is (a2) that is cut off 1/2 on the left, (e2) is (a2) that is maliciously falsified in the center by another image.



**Figure 9.** Anti-cutting Experiment (Lena)



**Figure 10.** Extracted-Watermark (Lena)



**Figure 11.** Extracted Watermark under cutting attack

In Figure 10, (a1) is extracted watermark from Figure 9(a1), (b1) is extracted watermark from Figure 9(b1), (c1) is extracted watermark from Figure 9(c1), (d1) is extracted watermark from Figure 9(d1), and (e1) is extracted watermark from Figure 9(e1).

In Figure 10, (a2) is extracted watermark from Figure 9(a2), (b2) is extracted watermark from Figure 9(b2), (c2) is extracted watermark from Figure 9(c2), (d2) is extracted watermark from Figure 9(d2), and (e2) is extracted watermark from Figure 9(e2).

In Figure 11, the extracted watermarks-- (a1), (b1), (c1), (d1) -- from Figure 9(e1) are obtained respectively according to the embedded watermark many times by using the self-similarity of Hilbert fractal curve. Likewise, the extracted watermarks-- (a2), (b2), (c2), (d2) -- from Figure 9(e2) are obtained

From Figure 10 and Figure 11, the blind extraction of watermark is realized respectively under the circumstances of no attacking, cutting off 1/8, cutting off 1/4, cutting off 1/2, and maliciously falsifying. And from Figure 10 and Figure 11, it can be concluded that the watermark's robustness with using Sierpinski carpet mask has stronger robustness than one without Sierpinski carpet mask under cutting and falsifying attack.

The experiment indicates good subjective and objective quality as well as the good imperceptibility and robustness of watermark as a result of using the Zigzag coding scrambling technology in the pre-

processing stage of watermark image and by using the self-similarity of Hilbert curve and Sierpinski carpet mask in watermark embedding.

## 5. Conclusion

An algorithm for embedding a meaningful 2-bit watermark image in carrier image is proposed by using Zigzag image scrambling based on discrete wavelet transform (DWT) and fractal self-similarity, along the Hilbert fractal curve scan sequence in the intermediate frequency subimage of wavelet decomposition of a carrier image, according to the mask template of Sierpinski carpet. Using Sierpinski carpet mask can ensure the good image quality of the central and sub-central parts of the watermark-embedded image as a result that the watermark information is not embedded into the central and sub-central parts of the carrier image. Extracting the embedded watermark does not need the original carrier image and the original watermark image, which is a blind watermark algorithm. By using fractal self-similarity and Sierpinski carpet mask, watermark information can be embedded into the non-central parts of the carrier image repeatedly  $N$  times. The algorithm has shown the higher watermark imperceptibility and robustness under cutting and falsifying attacks of image, which is proved to be true by experimental results. The algorithm can be applied in DRM to protect digital image copyright.

Using Sierpinski carpet mask can ensure the good image quality of the central and sub-central parts of the watermark-embedded image, but it should be more accurate that the watermark is avoided to be embedded into the Region of Interest (ROI) of the carrier image. In future research, the watermark-embedding algorithm based on fractal mask and ROI will be explored in order to improve the watermark imperceptivity and robustness.

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